Skew Dice™ Statistics Activities and Worksheets
written by Robert Fathauer

Teachers are given permission to print as many copies of the worksheets as needed for their own class. Individuals are given permission to print a copy for personal use.

Activity 1. Gathering and Plotting Data

Materials: Copies of Worksheet 1. Skew Dice™ and standard dot dice.

Objective: Learn to plan out how to gather experimental data, record the data, and create histograms from it.

Vocabulary:

Specific Common Core State Standards for Mathematics addressed by the activity:
Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. (7.SP 6)
Represent data with plots on the real number line. (S-ID 1)

Activity sequence:
Divide the students into ten groups. Give each group one Skew Die™ and one standard dot die, and give each student a copy of Worksheet 1.

1. Ask the students the probability of rolling a given number, such as six, with a standard dot die. Then ask how frequently a given number will come up on average.
Assuming the die is fair the probability is 1/6 for each number, and a given number will turn up on average every six rolls.

2. Note the assumption of the standard dot die being "fair". Discuss what fair means in a quantitative fashion. Ask the students if they think Skew Dice™ are as fair as standard dot dice.

3. Explain to the students that they will roll each type of die to gather data on what numbers turn up. Tell them each small group should first decide, after a couple of test rolls, the ground rules on valid data. E.g., should books or other barriers be used to keep a die from rolling off the desk or table? If a die does roll off onto the floor, do you count the value it lands on or reroll it? Does a cast die need to tumble some minimum number of times or distance in order to be counted? Should the die always be shaken around in the hand before making a casting motion?

4. Each group should roll each type of die thirty times. The students should take turns rolling the dice, with each student recording a tick mark in the appropriate box on his or her worksheet for each roll.

5. The students should plot the data for each type of die as a histogram on the worksheet.

6. Ask the students what the histogram is expected to look like for a fair die.
For fair dice, the histogram should ideally be flat.

7. Add the data for all ten groups together to form histograms for each type of die for the entire class (300 rolls per die if each group rolled them 30 times each). Does the class histogram look more like the expected fair dice histogram than the individual group histograms. Why or why not? Based on the histogram shapes, do the Skew Dice™ look approximately as fair as the standard dot dice?
An individual group's thirty rolls is really not enough data on which to draw conclusions, but the class's 300 rolls should be reasonably flat. If one or more numbers predominates, the dice may not be fair.
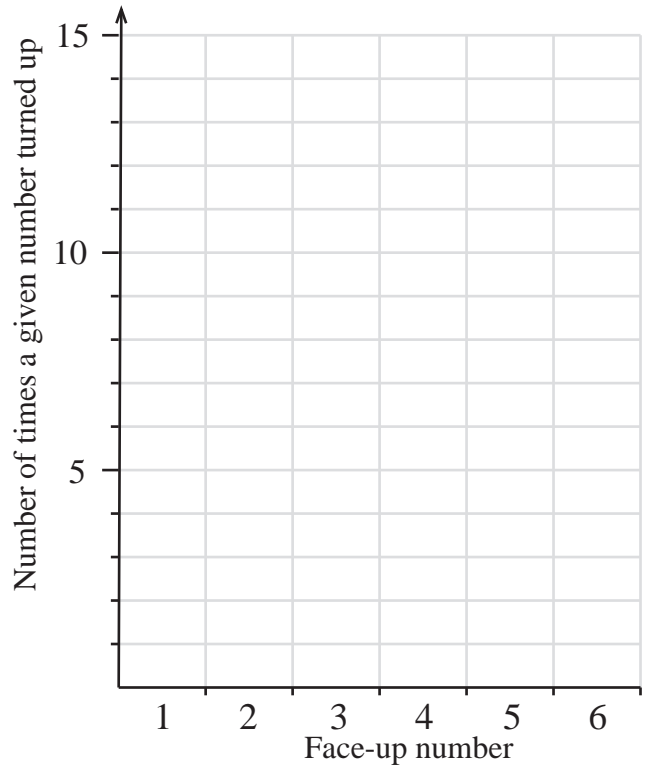
Worksheet 1. Gathering and Plotting Data

Notes on dice rolling methodology:

Rolled Number	1	2	3	4	5	6
Standard Dot Die						
Skew Die™						
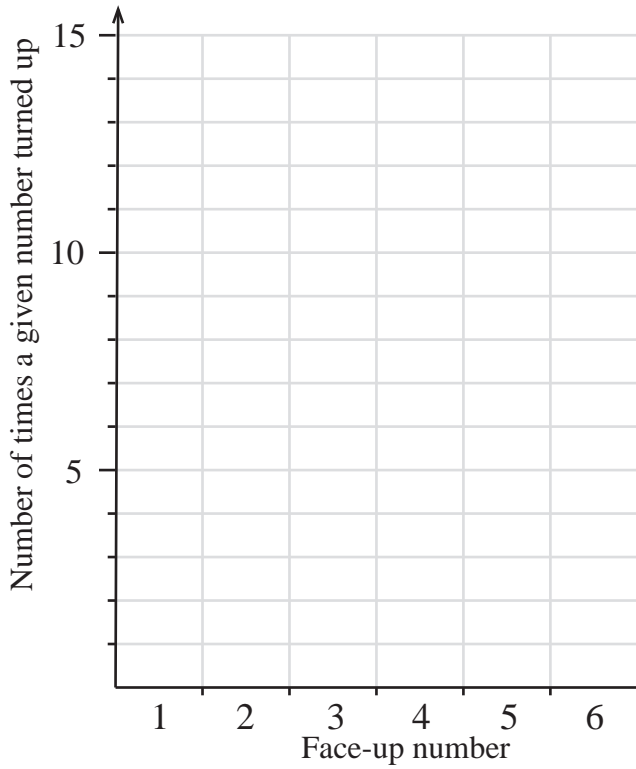
Standard Dot Dice

Skew Dice™

Activity 2. Mean, Median, Interquartile Range, Variance, and Standard Deviation

Materials: Copies of Worksheet 2. Completed Worksheets 1 from Activity 1.

Objective: Learn to calculate and interpret various statistical measures for characterizing data sets.

Vocabulary: Mean, Median, Mode, Interquartile Range, Variance, Standard Deviation

Specific Common Core State Standard for Mathematics addressed by the activity:
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. (S-ID 2)
Interpret differences in shape, center, and spread in the context of the data sets ... (S-ID 3)

Activity sequence:
Pass out copies of Worksheet 2. For a discussion of these statistical measures, it's best to use data that is expected to conform to a normal distribution. In Worksheet 1, histograms were generated that would ideally be flat. The data gathered in that exercise is plotted in this activity in a way that is expected to follow a normal distribution.

1. Discuss the meaning of the terms mean, median, and mode, being sure the distinction between the two is clear. Ask the students what the expected mean and median are for a fair die.

2. Using the data from Worksheet 1, have each student complete the table for number of sixes rolled by each group.

3. Have the students create a histogram for each type of die using this data. Discuss the shape of the histograms and how the shape indicates that a normal distribution would likely be appropriate as a model for the data.

4. Have the students write out the data for each die as a string of thirty integers, in increasing order. Use the data in this form to determine the median value.

5. Discuss the meaning of the term interquartile range.

6. Have the students mark the first and third quarters for the data written as a string of numbers. They should then calculate the interquartile range for each die.

7. Have the students compare the median, mean, and mode for each type of die. Do the values support using the normal distribution to characterize the data?
Ideally, these three numbers will be the same for data that conforms to a normal distribution. For the sample size used here, there will be differences, but all of the numbers are likely to be in the range 4-6.

8. Discuss the meaning of the terms variance and standard deviation.

9. Have the students calculate the variance and standard deviation for each die by using the values in the table at the top of the page.

10. Compare the standard deviation results from the different groups for the two types of dice.

Worksheet 2. Mean, Median, Interquartile Range, Variance, and Standard Deviation
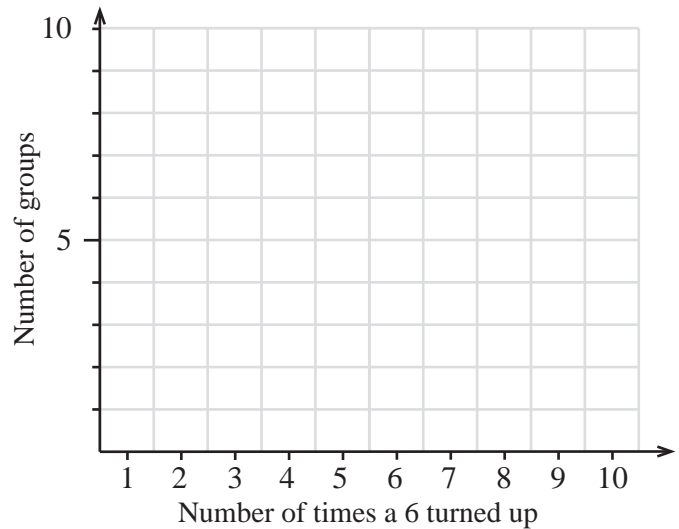
Using the data each group collected and recorded on Worksheet 1, fill out the table below for the number of sixes rolled by each group for each die.

Number of sixes rolled with:	Group number									
	1	2	3	4	5	6	7	8	9	10
Standard Dot Die										
Skew Die™										

.

Plot this data as histograms for each die:
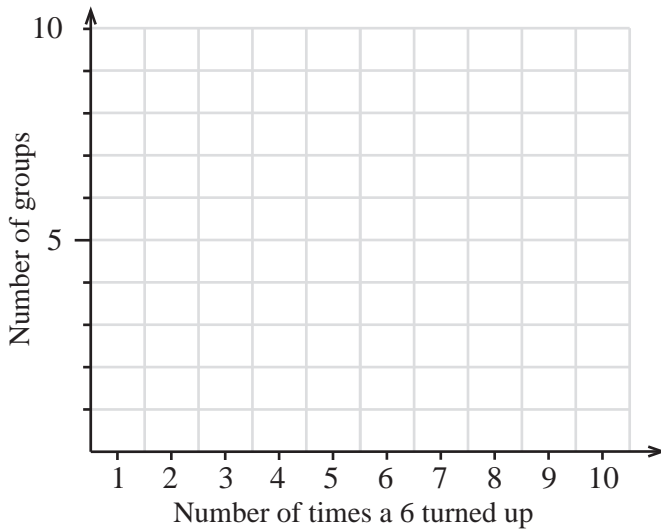
Standard Dot Dice

Skew Dice™

Looking at the shape of the histograms, does the data appear to conform to a normal distribution? Why or why not?

Write out the results from the table for each type of die as a string of numbers, in order of increasing value:

Standard Dot Die:

Skew Die™:

On the strings of numbers above, mark the medians. Write the values here:

On the strings of numbers above, mark the first and third quartiles. Calculate the interquartile range for each die and write the values here:

What is the mean and mode of the data for each die?

Compare the mean, median, and mode. Do the values support using a normal distribution to model the data?

Calculate the variance of the data for each die:

Use the variance to calculate the standard deviation of the data for each die:

Activity 3. Using the Normal Distribution to Analyze Data

Materials: Copies of Worksheet 3. Completed Worsksheets 2.

Objective: Learn to plot a normal distribution curve over experimental data and use such curves to calculate cumulative probabilities.

Vocabulary: Normal Distribution, Probability Density Function (PDF), cumulative probabilities.

Specific Common Core State Standard for Mathematics addressed by the activity:
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages ... Use calculators, spreadsheets, and tables to estimate areas under the normal curve. (S-ID 4)

Activity sequence:
Pass out the worksheets.

1. Describe the Probability Density Function for the normal distribution and make sure the students understand the formula.

2. Have the students fill out the table.

3. Have the students plot the points on the histograms of Worksheet 2 and sketch a bell curve passing through those points.

4. Discuss the fit of the data to the bell curves.

5. Explain what a cumulative probability is. Describe how the area under the curve can be used to determine cumulative probabilities. Have the students answer the questions about the standard normal distribution.

6. Be sure the students understand how to standardize the class data in order to use the table with the class values of mean and standard deviation from Worksheet 2.

7. Discuss the class results obtained over the three activities and what they reveal about the fairness of the two types of dice.

8. Ask the students why they think dice might not be perfectly fair. Some considerations and discussion points follow:
• In the real world, nothing is mathematically precise. Dice are made by injecting plastic into molds, and molds are never perfectly accurate. (Injection molding is an important industrial process, which could be described if desired.)
• After forming in the mold, dice are tumbled to smooth the surfaces, partly to remove nubs left where the plastic was injected. This tumbling process could introduce additional small variations in the shape.
• There are different numbers of dots on different faces, and there are pits where the dots are formed. This means the weight of the dice near the different faces is not the same. If this is an important factor, how would you expect it to show up in the data?
• It's possible that most of the dice of a given type are reasonably fair, but an occasional unfair die could result from a manufacturing flaw. E.g., there could be an air bubble in the plastic near one of the faces.
• The way in which the dice are rolled could prevent them from behaving fairly. In order to land on a random number, the rolled die needs to sample many possible states as it tumbles.
• Regular dice and Skew Dice™ are both based on isohedra. An isohedron is a polyhedron for which the symmetry group of the polyhedron is trasitive on the faces. This basically means that were it not for the dots there would be no way to distinguish any one face from any other. The polyhedron on which Skew Dice™ are based is called an asymmetric trigonal trapezohedron. (A cube is a special case of a symmetric trigonal trapezohedron.) While both shapes are isohedra, the cube may tumble more readily than the asymmetric trigonal trapezohedron, which could better facilitate fair tosses.

Worksheet 3. Using the Normal Distribution to Analyze Data

The normal distribution describes a data distribution that is observed in a wide variety of situations. It produces a bell-shaped curve that is specified by the mean μ and standard deviation σ of the data. The probability Density Function (PDF) for the normal distribtion is given by the following:

$$PDF = \frac{1}{\sigma\sqrt{2\pi}}\, e^{-(x-\mu)^2/(2\sigma^2)}$$

Note that this function is at a maximum when $x = \mu$ and is symmetric about μ. Using a calculator and the values for μ and σ from Worksheet 2, fill out the following table. A capital Z is often used to denote the number of standard deviations from the mean.

$x - \mu$	0	σ (Z = 1)	-σ (Z = -1)	2σ (Z = 2)	-2σ (Z = -2)	3σ (Z = 3)	-3σ (Z = -3)
x							
PDF (standard die)							
PDF (Skew Die™)							

Plot these points on the histograms of Worksheet 2 for each die and sketch a bell curve passing through the points. In that case, the area of the bars sums to the number of data points, 10. The above formula is normalized such that the area under the curve integrates to 1. To overlay the points above on the histogram, you should multiply all of the values by 10.

How well does the data fit the curves?

The area under the curve can be used to determine cumulative probabilities. Calculation of the area under the curve can be done using computer programs, an advanced calculator, or tables such as the one reproduced on the accompanying sheet. Use the table to answer the following questions.

For the standard normal distribution, what is the probability that a measured value for the quantity x is greater than 0?

For the standard normal distribution, what is the probability that a measured value for the quantity x lies between 0 and 1?

For the standard normal distribution, what is the probability that a measured value for the quantity x lies between 0 and -2?

For the standard normal distribution, what is the probability that a measured value for the quantity x lies between 1 and 3?

Answer the following questions for your class data for the number six turning up (the x value), from Worksheet 2. Use the formula on the table page to standardize your class data.

Ideally, for a fair die rolled 30 times, the number 6 would turn up 5 times. This means the cumalative probability above or below 5 would be 0.5. For your class data, what is the cumulative probability for x being greater than 5 for standard dot dice?

What is the cumulative probability for x being greater than 5 for Skew Dice™?

What is the cumulative probability for x lying between 6 and 7 for standard dot dice?

What is the cumulative probability for x lying between 0 and 3 for Skew Dice™?

Based on your class results and analysis, do you think the standard dot dice are really fair? Try to quantify your answer as much as possible.

Cumulative Distribution Function of the Standard Normal Distribution

The table below gives the area under the standard normal distribution curve (mean of 0 and standard deviation of 1), as shown at right. The shaded area represents the probability that a measurement x will fall between 0 and a.

The table can be used with a non-standard normal distribution by standarizing your values. If you want to find the cumulative distribution with a mean of μ and a standard deviation of σ, then use $|\mu - a|/\sigma$ for the x value in the table.
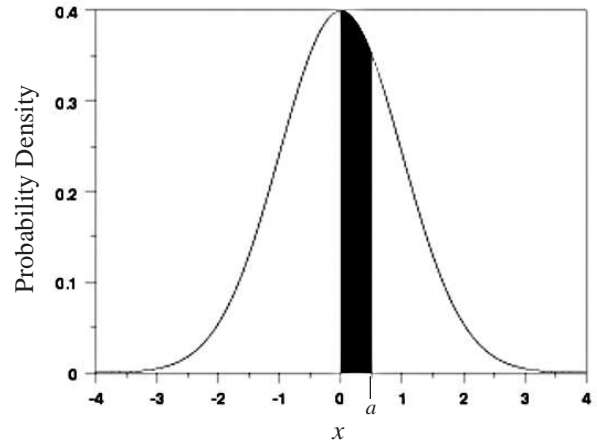
Area under the Normal Curve from 0 to X

X	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.00000	0.00399	0.00798	0.01197	0.01595	0.01994	0.02392	0.02790	0.03188	0.03586
0.1	0.03983	0.04380	0.04776	0.05172	0.05567	0.05962	0.06356	0.06749	0.07142	0.07535
0.2	0.07926	0.08317	0.08706	0.09095	0.09483	0.09871	0.10257	0.10642	0.11026	0.11409
0.3	0.11791	0.12172	0.12552	0.12930	0.13307	0.13683	0.14058	0.14431	0.14803	0.15173
0.4	0.15542	0.15910	0.16276	0.16640	0.17003	0.17364	0.17724	0.18082	0.18439	0.18793
0.5	0.19146	0.19497	0.19847	0.20194	0.20540	0.20884	0.21226	0.21566	0.21904	0.22240
0.6	0.22575	0.22907	0.23237	0.23565	0.23891	0.24215	0.24537	0.24857	0.25175	0.25490
0.7	0.25804	0.26115	0.26424	0.26730	0.27035	0.27337	0.27637	0.27935	0.28230	0.28524
0.8	0.28814	0.29103	0.29389	0.29673	0.29955	0.30234	0.30511	0.30785	0.31057	0.31327
0.9	0.31594	0.31859	0.32121	0.32381	0.32639	0.32894	0.33147	0.33398	0.33646	0.33891
1.0	0.34134	0.34375	0.34614	0.34849	0.35083	0.35314	0.35543	0.35769	0.35993	0.36214
1.1	0.36433	0.36650	0.36864	0.37076	0.37286	0.37493	0.37698	0.37900	0.38100	0.38298
1.2	0.38493	0.38686	0.38877	0.39065	0.39251	0.39435	0.39617	0.39796	0.39973	0.40147
1.3	0.40320	0.40490	0.40658	0.40824	0.40988	0.41149	0.41308	0.41466	0.41621	0.41774
1.4	0.41924	0.42073	0.42220	0.42364	0.42507	0.42647	0.42785	0.42922	0.43056	0.43189
1.5	0.43319	0.43448	0.43574	0.43699	0.43822	0.43943	0.44062	0.44179	0.44295	0.44408
1.6	0.44520	0.44630	0.44738	0.44845	0.44950	0.45053	0.45154	0.45254	0.45352	0.45449
1.7	0.45543	0.45637	0.45728	0.45818	0.45907	0.45994	0.46080	0.46164	0.46246	0.46327
1.8	0.46407	0.46485	0.46562	0.46638	0.46712	0.46784	0.46856	0.46926	0.46995	0.47062
1.9	0.47128	0.47193	0.47257	0.47320	0.47381	0.47441	0.47500	0.47558	0.47615	0.47670
2.0	0.47725	0.47778	0.47831	0.47882	0.47932	0.47982	0.48030	0.48077	0.48124	0.48169
2.1	0.48214	0.48257	0.48300	0.48341	0.48382	0.48422	0.48461	0.48500	0.48537	0.48574
2.2	0.48610	0.48645	0.48679	0.48713	0.48745	0.48778	0.48809	0.48840	0.48870	0.48899
2.3	0.48928	0.48956	0.48983	0.49010	0.49036	0.49061	0.49086	0.49111	0.49134	0.49158
2.4	0.49180	0.49202	0.49224	0.49245	0.49266	0.49286	0.49305	0.49324	0.49343	0.49361
2.5	0.49379	0.49396	0.49413	0.49430	0.49446	0.49461	0.49477	0.49492	0.49506	0.49520
2.6	0.49534	0.49547	0.49560	0.49573	0.49585	0.49598	0.49609	0.49621	0.49632	0.49643
2.7	0.49653	0.49664	0.49674	0.49683	0.49693	0.49702	0.49711	0.49720	0.49728	0.49736
2.8	0.49744	0.49752	0.49760	0.49767	0.49774	0.49781	0.49788	0.49795	0.49801	0.49807
2.9	0.49813	0.49819	0.49825	0.49831	0.49836	0.49841	0.49846	0.49851	0.49856	0.49861
3.0	0.49865	0.49869	0.49874	0.49878	0.49882	0.49886	0.49889	0.49893	0.49896	0.49900
3.1	0.49903	0.49906	0.49910	0.49913	0.49916	0.49918	0.49921	0.49924	0.49926	0.49929
3.2	0.49931	0.49934	0.49936	0.49938	0.49940	0.49942	0.49944	0.49946	0.49948	0.49950
3.3	0.49952	0.49953	0.49955	0.49957	0.49958	0.49960	0.49961	0.49962	0.49964	0.49965
3.4	0.49966	0.49968	0.49969	0.49970	0.49971	0.49972	0.49973	0.49974	0.49975	0.49976
3.5	0.49977	0.49978	0.49978	0.49979	0.49980	0.49981	0.49981	0.49982	0.49983	0.49983
3.6	0.49984	0.49985	0.49985	0.49986	0.49986	0.49987	0.49987	0.49988	0.49988	0.49989
3.7	0.49989	0.49990	0.49990	0.49990	0.49991	0.49991	0.49992	0.49992	0.49992	0.49992
3.8	0.49993	0.49993	0.49993	0.49994	0.49994	0.49994	0.49994	0.49995	0.49995	0.49995
3.9	0.49995	0.49995	0.49996	0.49996	0.49996	0.49996	0.49996	0.49996	0.49997	0.49997
4.0	0.49997	0.49997	0.49997	0.49997	0.49997	0.49997	0.49998	0.49998	0.49998	0.49998