Activity 4.1. Surface areas of Archimedean solids

Materials: Worksheet 4.1 (transparency if using overhead projector) and copies for students.

Objective: Learn to calculate the surface areas of polyhedra by calculating the areas of the individual faces and summing over all the faces.

Vocabulary: Archimedean solid, face, vertex, edge, cuboctahedron, truncated octahedron, truncated cube

Specific Common Core State Standard for Mathematics addressed by the activity:
Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. (7.G 6)

Activity Sequence:
1. Pass out the worksheet.

2. Write the vocabulary terms on the board and discuss the meaning of each one.
Definitions of vocabulary terms may be found in the glossary at the back of the book.

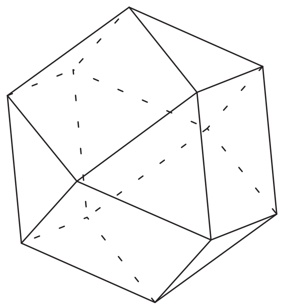
3. Have the students perform the first task on the worksheet. This requires using the formula for the area of a triangle in terms of its base and height, and then adding the areas of the individual faces. Ask a student to share his or her result and to describe how it was calculated.
$6 + 2\sqrt{3}$.

4. Have the students perform the second task on the worksheet. This involves calculating the area of a regular hexagon. The easiest way to do this is to recognize that the hexagon can be divided into six equilateral triangles. Ask a student to share his or her result and to describe how it was calculated.
$6 + 12\sqrt{3}$.

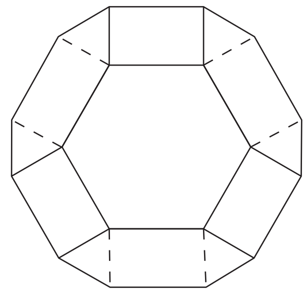
5. Have the students use their calculators to get an approximate ratio of the area of these two polyhedra.
A truncated octahedron has a surface area approximately 2.83 times that of a cuboctahedron with the same edge lengths.

6. Have the students perform the third task on the worksheet. This involves calculating the area of a regular octagon. Ask a student to share his or her result and to describe how it was calculated.
A regular octagon with unit edge length has an area of $2 + 2\sqrt{2}$. The total surface area of the truncated cube is $12 + 12\sqrt{2} + 2\sqrt{3}$. The cube has surface area $6(1 + \sqrt{2})^2 = 18 + 12\sqrt{2}$. The ratio of the surface area of the truncated cube to that of the cube is approximately 0.927.

Worksheet 4.1. Surface areas of Archimedean solids



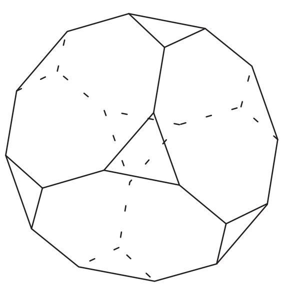
Cuboctahedron

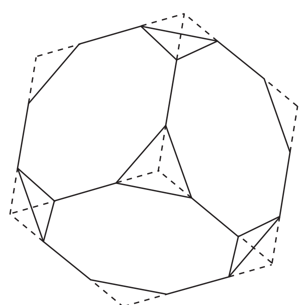


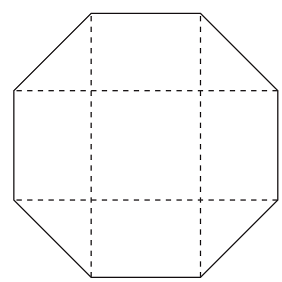
Truncated Octahedron

Calculate the surface area of a cuboctahedron with edges of length 1. The height of an equilateral triangle with base equal to 1 is $\sqrt{3}/2$.

Calculate the surface area of a truncated octahedron with edges of length 1. How do the surface areas of these two polyhedra compare?







Truncated Cube

Calculate the surface area of a truncated cube with edges of length 1. The diagram at right will help you calculate the area of a regular octagon. If the truncated cube was formed by removing the corners of a cube as shown, how do the surface areas of the two solids compare?